HYDRODYNAMIC EFFECTS IN PHYSICAL-CHEMICAL MECHANICS OF FRACTURE

Malkin A. I.

A.N. Frumkin Institute of PhysicalChemistr and Electrochemisty, RAS, Moscow

In the last few decades considerable advances have been made in the revealing of kinetic regularities of environment induced crack growth. The problem has been studied both from a theoretical point of view and by means of direct experiments [1,2]. Nevertheless, the present notion on the rate-determining processes is far from being complete. Two aspects should be recognized in theoretical studies of environment – induced cracking. The first one is concerned with micromechanisms of crack growth, i.e. with the description of the fracture elementary event and the preceding processes localized at the crack tip. The second one involves an adequate description of transport of the active environment to a crack tip. Although the first part of the problem are the most incomprehensible and important (at least, from chemical point of view), some of the experimentally observed regularities seem to be dictated by the transport of an environment.

Transport processes, as applied to environment-induced fracture, have been discussed for the cases of liquid metal embrittlement (LME), stress corrosion and surface crack growth under the influence of cutting fluids [1–4]. A tentative analysis within the context of a self-consistent scheme has been made in [5].

Here the object is to evaluate the role of nonlocal hydrodynamic interaction between the solid and environment in crack kinetics. On the basis of experimentally observed regularities, semiempirical models of liquid environment – induced crack propagation are suggested. Two possible rate-determining mechanisms stemming from nonlocal interaction are considered: the first is the cavitation one (i.e. the loss of continuity of a liquid) and the second is the reduction of local stress at the crack tip.

In the majority of kinetic experiments, the quantity under measurement is time – dependent crack length. The results are usually reported in terms of so-called kinetic diagrams (KD) which present the relationship between the crack growth rate and stress intensity factor (SIF) at given temperature, pressure and chemical composition of the environment.

Fig. 1 shows the representative form of KD [1, 6].

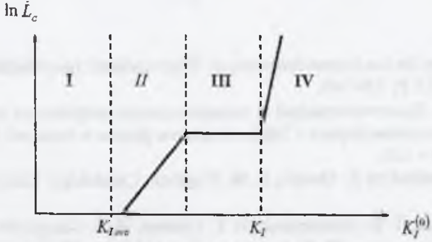

Fig. 1. Typical shape of kinetic diagram.

The SIF – dependence of crack growth rate generally incorporates four clearly defined regions: region I is the zone of subliminal values of SIF corresponding to the lack of crack growth; region II is located to the right of threshold value of SIF K_{Iscc} and corresponds to environment – induced growth of a crack; region III is characterized by a practically constant growth rate; finally, region IV is representative of the stage of crack growth without the essential participation of an environment. It is commonly supposed that the rate-determining processes at the stages II and III are respectively environment – induced fracture and transport of an environment to a crack tip.

The temperature dependence of the crack growth rate for each of the stages II...IV is normally expressible as Arrhenius law. As this takes place, the activation energy at the stage III is much less than that at the II and the more so the IV ones. This distinction indicates the change of rate – determining process with increasing SIF.

In this context it should be recalled that representation of experimental data in terms of KD, being convenient, does not reflect the actual condition at the crack tip. The construction of KD is performed at given external conditions, whereas the processes responsible for crack growth depend on the local ones in the vicinity of a crack tip. External conditions affect the crack growth rate, at most, by the indirect route. Thermodynamic parameters and even phase condition of a fluid at the crack tip can be significantly different from the external ones. Furthermore, in the construction and inspection of KD the difference between actual and "geometrical" value of SIF (this latter one is calculated with unloaded crack faces) are ignored, whereas there exists some evidence that the difference is quite essential.

As an example one can refer to experimental results on crack propagation in LME [1]. It has been found that the growth rate at the steady-rate stage III strongly depends on the external load. To put it in another way, the growth rate does not depend on the crack length and, hence, on the local stress calculated for the load-free crack faces. At the same time, the growth rate depends heavily on the stress far off the crack tip. The facts of such kind seem to be first recorded in early studies of LME [4]. The above facts are clearly indicative of an important role of solid-environment interaction out of the crack tip. As is shown below, these special features of crack evolution are likely caused by the reduction in SIF and/or the loss of continuity of a liquid stemming from the pressure drop inside the opening and/or growing crack.

Theoretical studies of hydrodynamic effects in crack kinetics are based on extremely simplified semiempirical models [5]. We consider a surface crack of length $L_c(t)$ in a half-space $x > 0$ bordering with incompressible Newtonian fluid. Let the half-space at $y \to \pm \infty$ be loaded by tensile stress σ_∞. A number of not-too-exacting restrictions enables one to use the approximation of inertialess viscous flow with local velocity profile approaching the Poiseulle's one. As a consequence the cross-average velocity of fluid flow $u(x,t)$ and the pressure gradient along the crack growth direction are related by the equation

$$u(x,t) = -\frac{h^2}{3\mu}\frac{\partial P}{\partial x}, \qquad (1)$$

where μ is fluid viscosity. The displacement of the crack faces $h(x,t)$ satisfies the continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0 \qquad (2)$$

resulted from the averaging of continuity equation for incompressible fluid over the cross-section of a crack. Eqs. (1) and (2) are supplemented by the specification of external pressure P_0 and the adhesion (impermeability) condition at the crack tip.

To relate the crack opening displacement to liquid pressure, a crack model has to be specified. For this purpose one can use the well known integro-differential relations of Barenblatt-Dugdale model [2]. The quasi-brittle character of environment-induced fracture however enables us to consider the simplified approach which is not associated with specific models of the plastic zone. We replace the crack opening displacement of Eq. (1) and (2) by its asymptotic relation

$$h \cong \frac{2\left(1-\nu_p\right)K_I}{\sqrt{2\pi}\,G}\sqrt{L_c+R_p-x}$$

$$R_p \cong \frac{\pi}{8}\left(\frac{K_I}{\sigma_s}\right)^2 \qquad (3)$$

where K_I is the stress intensity factor, $\sigma_s \equiv const$ is the effective yield stress limit, G is the shear modulus, ν_p is Poisson's ratio and R_p is the size of plastic zone. That simplification is somewhat justified by the fact that the absolute value of fluid pressure peaks in a crack tip. Outside the vicinity of a crack tip the pressure variation is relatively small. It is then hoped that x-dependence of pressure is scarcely affected by the error in crack opening displacement.

The use of Eq. (3) instead of integro-differential relations essentially facilitate the problem. Given crack opening displacement as defined by Eq. (3) with beforehand unknown time-dependent SIF, fluid velocity is determined by a straightforward integration of Eq. (2). Thereafter the fluid pressure is calculated by integration of Eq. (1).

To relate the distribution of fluid pressure to the magnitude of SIF, we invoke the approximate expression [7] to give

$$K_I - K_I^{(0)} = -3\sqrt{\pi}c_1\frac{\mu G^2\sqrt{L_c}\,\dot{L}_c}{\left(1-\nu_p\right)^2 K_I^2} - 2\sqrt{\pi}c_2\frac{\mu G^2 L_c^{3/2}\dot{K}_I}{\left(1-\nu_p\right)^2 K_I^3}, \qquad (4)$$

where the time derivatives is labeled by the points at the top, $K_I^{(0)} \equiv 1.12\sqrt{\pi L_c}\,\sigma_\infty$; $c_1 = 1.51$; $c_2 = 1.07$. With the same accuracy the fluid pressure at the crack tip can be calculated from the relationship

$$P_c - P_0 = \frac{3\pi\mu\,G^2\dot{L}_c}{2\,(1-\nu_p)^2 K_I^2}\ln\frac{R_p}{L_c} - \frac{\pi\mu\,G^2 L_c\dot{K}_I}{(1-\nu_p)^2 K_I^3}. \qquad (5)$$

With the adopted semiempirical approach Eqs. (1)–(5) should be supplemented with the relation for crack growth rate as a function of SIF, pressure P_c and temperature T_c of a fluid in the vicinity of a crack tip. The consideration below is restricted by isothermal problems so that T_c is taken to be equal to an external temperature. The role of temperature increasing seems to be of relatively little importance in most cases.

As a consequence the system of ordinary differential equations descriptive of the fluid-filled crack growth can be rewritten in the ultimate form

$$\dot{K}_I + \frac{3c_1}{2c_2}\frac{v_0 K_I}{L_c}f\left(K_I, P_c, T\right) + \frac{\left(1-\nu_p\right)^2 K_I^3\left(K_I - K_I^{(0)}\right)}{2\sqrt{\pi}c_2\mu\,G^2 L_c^{2/3}} = 0, \qquad (6)$$

$$\dot{L}_c = v_0\,f\left(K_I, P_c, T\right)$$

where P_c is determined by Eq. (5).

The distinction between function $f(K_I, P_c, T)$ and experimentally obtained KD (Fig. 1) is that the former incorporates the actual value of SIF depending on the fluid pressure distribution inside a crack and fluid conditions at the crack tip, whereas the latter is defined as a function of "geometrical" value of SIF and fluid conditions outside the crack.

To complete the above models we need to specify function $f(K_I, P_c, T)$ relating the growth rate to SIF and fluid pressure at the crack tip. Let the impact of nonlocal "liquid-solid" interaction on the crack growth rate be small. Then first order corrections to SIF and pressure stemming from Eqs. (4) and (5) are given by the following expressions:

$$\Delta K_I \cong -12{,}5\sqrt{\pi}\,\frac{\mu G^2 \sqrt{L_c}\,\dot{L}_c}{\left(1-\nu_p\right)^2 K_I^{(0)2}},$$

$$\Delta P_c \cong -\frac{2{,}68\,\pi\mu\, G^2 \dot{L}_c}{(1-\nu_p)^2 \sigma_\infty^2 L_c}\ln\frac{\sigma_s}{\sigma_\infty}\,. \tag{7}$$

As evident from Eq. (7), in the event of sufficiently strong SIF-dependence of function $f(K_I, P_c, T)$, nonlocal hydrodynamic effects have to be more pronounced with lengthy cracks. At the initial stage of growth, while the crack length is small, these effects are of little significance. Hence, for the SIF-dependence of functions $f(K_I, P_c, T)$ in Eq. (6) one can conceptually use the empirical interrelations approximating the growth rate at the stage II of KD Fig. 1.

The question on pressure-dependence of functions $f(K_I, P_c, T)$ is more complicated. The matter is that to complete the models we need to have these relations for a negative pressure range corresponding to metastable state of a liquid. The experiments under these conditions seem to be too difficult to be practical. As for the "microscopic" models, the question on the pressure-dependence of crack growth rate appears to be yet untouched. Nevertheless, there exist the situations that the role of that factor is moderately important. This is particularly true when the cavitation microcavities originates before the significant pressure-related variation in the growth rate. Below we assume functions $f(K_I, P_c, T)$ are independent of the pressure at the crack tip.

In the subsequent consideration we use exponential or threshold power-type functions [1,2]

$$\dot{L}_c = \nu_0 \exp\left(-\frac{E - bK_I}{kT}\right), \tag{8}$$

$$\dot{L}_c = \nu_0\left(\frac{K_I}{K_{Iscc}} - 1\right)^n \eta\left(\frac{K_I}{K_{Iscc}} - 1\right)\exp\left(-\frac{E}{kT}\right). \tag{9}$$

One necessary remark needs to be made here. Since we assume the initial crack is completely filled with a liquid, the initial crack opening displacement have to be nonzero. It can be clearly the case only with nonzero SIF and, hence, with nonzero initial stress $\sigma_\infty^{(0)}$. This corresponds to situation of provisional wetting of a solid under the small subliminal loading. An alternative treatment of the quantity $\sigma_\infty^{(0)}$ relates it with the initial value of crack tip opening displacement. The nonzero initial stress in that instance is introduced to accommodate roughly the residual stress at the crack tip remaining of the creation of artificial crack (notch). With the adopted deterministic approach the crack initiates as soon as SIF ranges up to the threshold value. To consider the problem on crack starting one should set $L_c = 0$ in Eq. (6). The condition of starting is therewith written, in terms of Eq. (8), as $K_I = K_{Iscc}$.

We defined the delay time of crack starting as the time SIF takes to attain a threshold value under the instantaneous loading from small subthreshold stress $\sigma_\infty^{(0)}$ to some super-threshold one σ_∞ exceeding the threshold external stress. The delay time t_s is expressible as an expansion in terms of small parameters $\sigma_\infty^{(0)} / \sigma_\infty$ and $\pi L_c \sigma_\infty^{(0)2} / K_{Iscc}^2$. A reasonably accurate relation is given by the dominant part of the expansion:

$$t_s \cong \frac{0.88}{\pi} \frac{\mu G^2}{(1-\nu_p)^2 \sigma_\infty \sigma_\infty^{(0)2}} . \tag{10}$$

As evident from Eq. (10), the delay time to a first approximation depends neither on threshold value of SIF, nor on the initial crack length. That special feature is clearly dictated by strong cubic SIF-dependence of the rate of change of the SIF (see Eq. (6)). For this reason the most part of the delay time falls on the states with small SIF and crack opening displacement. The spasmodic increasing of SIF takes place at the very end of the delay period.

For the case of the loading with constant strain rate, the qualitative picture is just the same if only the strain rate is moderately low. As above, the prolonged stage of insignificant variation of SIF alternates with its spasmodic increasing to the actual "geometrical" value. While the strain rate meets the condition

$$\frac{\mu G^2 \dot{\varepsilon}}{(1-\nu_p)^2 \sigma_\infty^{(0)3}} \gg 1 \tag{11}$$

if is not difficult to obtain an asymptotic interrelation for the external stress of crack starting σ_g. Retaining only the dominant part of the asymptotic expansion, we have

$$\sigma_g \cong \begin{cases} \dfrac{0.89 K_{Iscc}}{\sqrt{\pi L_c(0)}}, & \dot{\varepsilon} < \dot{\varepsilon}_* \\ \dfrac{1.32 G}{\sqrt{\pi}(1-\nu_p)} \sqrt{\dfrac{\mu \dot{\varepsilon}}{\sigma_\infty^{(0)}}}, & \dot{\varepsilon} > \dot{\varepsilon}_* \end{cases} \tag{12}$$

where the critical value of the strain rate ε_* is given by the expression

$$\dot{\varepsilon}_* \cong \frac{0.46 (1-\nu_p)^2 K_{Iscc}^2 \sigma_\infty^{(0)}}{\mu G^2 L_c(0)} . \tag{13}$$

From the results obtained it follows that, with subcritical strain rates, the nonlocal effect of liquid environment on the starting stress is negligible. In contrast, with supercritical strain rates, that effect is found to be very important.

Nonlocal hydrodynamic interaction of liquid environment and stressed solid appears also to be essential for crack growth kinetics. Below the peculiarities of growth kinetics are considered for the situation of constant loading. Qualitative examination of system Eqs. (6) and (8) shows that, with the constraint

$$\beta = (1-\nu_p)^2 e^{E/kT} k^2 T^2 \sigma_\infty / \mu G v_0 b^2 > 9.02, \tag{14}$$

there exist qualitatively distinct regimes of crack growth, among them, the steady-rate ones. Depending on the initial condition SIF tends to some constant value with increasing length of a crack or, alternatively, to zero. In the considered problem the initial value of SIF is dictated by the previous stage of unsteady loading. Therefore, the result obtained implies that, with sufficiently great value of β, kinetic behavior of a crack can be qualitatively distinct depending upon the prehistory of a loading to the constant stress σ_∞. In particular, with an "instantaneous" provisional loading, the growth of initially lengthy crack will follow the steady-rate asymptotics with nonzero constant SIF, whereas the short cracks will decelerate.

It should be noted that the typical values of β presumably range from 10^5 to 10^{10}. Since $\beta >> 1$, the limiting value of SIF can be evaluated by the use of asymptotic expansion. As a consequence the crack growth rate at the steady rate stage takes the form

$$\dot{L}_c \cong 0.10 \frac{(1-\nu_p)^2 \sigma_\infty}{\sqrt{\pi\mu}} \left(\frac{E}{bG}\right)^2 \tag{15}$$

which is applicable in the case of $E/kT >> 1$.

The qualitative behavior of a crack within the model Eqs. (6) and (9) is quite similar to the above one. For the most interesting case $n > 2$, the length-dependence of crack growth rate flattens out or, alternatively, the growth is terminated.

In that case we obtain the following approximate relation for a growth rate at the steady-rate stage:

$$\dot{L}_c \cong \frac{K_{Iscc}^2}{\mu G} \left[\frac{0.10\,(1-\nu_p)^2\,\sigma_\infty}{G}\right]^{\frac{n}{n-2}} \left(\frac{K_{Iscc}^2}{\mu G v_0}\right)^{\frac{2}{n-2}} \exp\left[\frac{2E}{(n-2)kT}\right]. \tag{16}$$

This expression differs essentially from Eq. (15). The temperature-dependence of growth rate Eq. (15) is dictated by the activation energy of the fluid viscosity E_v which is generally small in comparison with "chemical" values of activation energy of disruption of interatomic bonds. The temperature-dependence of growth rate Eq. (16) corresponds to activation energy

$$E_c = \frac{nE_v - 2E}{n-2} \tag{17}$$

the value and even the sign of which depend on the exponent n in Eq. (9). Of course, that conclusion is valid in the case of weak temperature-dependence of the exponent and threshold value of SIF.

By this means the mechanism of reduction of local stress can be conceptually the reason of experimentally observed steady-rate growth of a crack. However, the above steady-rate regimes can be realized in the event that fluid continuity does not violate at the preceding stage of crack growth. On the other hand, the origination of microcavities at the vicinity of a crack tip may first occur at the steady-rate stage. In any case, the above discussed form of empirical KD points to the need for consideration of cavitation effects.

Since the maximum pressure drop is localized in small vicinity of a crack tip, it is in this domain one should expect the loss of continuity of a liquid. To account for the loss of an immediate liquid- solid contact, the boundary condition at the crack tip should be renewed. The adhesion condition for liquid velocity should be replaced by the evident condition for liquid pressure (we use the conventional notations and restrict the evaluation to the case of complete wetting)

$$P_c = P_V - \frac{\gamma_{LV}}{h_c} \tag{18}$$

The velocity of the liquid-vapor interface (LVI) has to be calculated now from Eqs. (1) and (2). At the initial stage of crack growth this velocity is greater than that of the crack tip. Therefore, the impermeability condition needs to be used. In the subsequent evolution the crack tip opening displacement however increases and the velocity of LVI is reduced. The condition of critical microcavity origination has to be thus formulated as the equality of LVI velocity and crack growth rate.

We assume that, without the immediate liquid- solid contact, environmental influence on a crack growth rate can be neglected. If this is the case, concurrent with the critical microcavity origination the crack tip velocity abruptly drops, and the contact will be reestablished. The growth rate has to be hence equal to the critical value.

This has to be the case as long as the critical rate is greater than that one without environmental effect. As soon as this condition is violated, the critical microcavity evolution doesn't result in decreasing of growth rate. Cavitation mechanism of growth stabilization becomes therefore invalid.

For the case of small reduction of local stress we obtain the following expression for the growth rate at the steady-rate stage:

$$\dot{L}_c \cong \frac{0{,}59}{\pi^2} \frac{\gamma_{LV}}{\mu\, G \ln(\sigma_s / \sigma_\infty)} \left[1 + \frac{(1-\nu_p) K_I^{(0)2} (P_0 - P_V)}{2\, \gamma_{LV} \sigma_s G} \right] \tag{19}$$

Quantitative evaluation with Eq. (19) shows that the factor in square brackets is close to unity for almost all interesting cases. By this means, the growth rate doesn't substantially depend on local stress. We can conclude that cavitation mechanism as well as the above mechanism of reduction of local stress brings into existence the steady-rate stage of crack growth.

References

1. C. L. Briant, S. K. Banerji (Eds), Embrittlement of Engineering Alloys, Academic Press, New York, 1983.
2. G. P. Cherepanov, Mechanics of Brittle Fracture, Nauka, Moscow, 1974 (in Russian).
3. R. C. Westwood, N. S. Stoloff (Eds), Environment-Sensitive Mechanical Behavior, Gordon and Breach Sci. Publ., New York – London – Paris, 1966.
4. W. Rostoker, J. M. McCaughy, H. Markus, Embrittlement by Liquid Metals, Reinhold Publ. Corp., New York, Chapman and Hall Ltd., London, 1960.
5. Malkin, E. M. Podgaetsky, Physics – Doklady, v. 43, N 1, (1998), 1-5.
6. S. Wiederhorn, J. Am. Cer. Soc., v. 55, N 2 (1972), 81-85.
7. D. P. Rooke, D. A. Jones, J. Str. Anal. Eng. Design, 14 (1979), 1-6.