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# MODEL OF FORMATION OF THE OPTIMAL INVESTMENT PORTFOLIO WITH A LIMITED BUDGET

# МОДЕЛЬ ФОРМИРОВАНИЯ ОПТИМАЛЬНОГО ПОРТФЕЛЯ ИНВЕСТИЦИЙ ПРИ ОГРАНИЧЕННОМ БЮДЖЕТЕ

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ABSTRACT

*MATHEMATICAL MODEL, INVESTMENT PORTFOLIO, NET PRESENT VALUE, PROFITABILITY INDEX, LINEAR PROGRAMMING, LINEAR OPTIMIZATION, SPREADSHEET PROCESSOR, SOLVER ADD-ON*

*The article discusses a mathematical model of the formation of an optimal investment portfolio by the criterion of maximizing the total net present value (NPV) with preliminary ranking by the profitability index (PI) in a limited budget.*

АННОТАЦИЯ

*МАТЕМАТИЧЕСКАЯ МОДЕЛЬ, ИНВЕСТИЦИОННЫЙ ПОРТФЕЛЬ, ЧИСТАЯ ПРИВЕДЕННАЯ СТОИМОСТЬ, ИНДЕКС РЕНТАБЕЛЬНОСТИ, ЛИНЕЙНОЕ ПРОГРАММИРОВАНИЕ, ЛИНЕЙНАЯ ОПТИМИЗАЦИЯ, ТАБЛИЧНЫЙ ПРОЦЕССОР, НАДСТРОЙКА ПОИСК РЕШЕНИЯ*

*В статье рассмотрена математическая модель формирования оптимального инвестиционного портфеля по критерию максимизации общей чистой приведенной стоимости (NPV) с предварительным ранжированием по индексу рентабельности (PI) в условиях ограниченного бюджета.*

In specialized economic literature, the concept of "investment portfolio" is widely used. On the one hand, this concept means a set of investment projects that should eventually bring a certain income after the initial investment of some funds. On the other hand, a set of securities – bonds, stocks, enterprise assets should also

provide future returns for the owner.

By I.A. Blank's definition [1], an investment portfolio is "a purposefully formed set of financial instruments intended for financial investment in accordance with the developed investment policy".

Optimization of the investment portfolio implies the achievement of investment goals by determining the ratio of individual investment objects, taking into account the available investment resources. Most often, investment optimization is performed according to such criteria as increasing the return on the investment portfolio and/or reducing the risks of the investment portfolio.

In addition, optimization is possible in order to change the number of elements in the investment portfolio either by ensuring the internal stability of the investment portfolio or by ensuring the growth of the company's capitalization. At the same time, the investment portfolio must correspond to the volume of investment resources that provide a certain level of profitability and portfolio liquidity, taking into account risks.

*The purpose* of this research is to solve the problem of choosing investment projects in conditions of a limited budget. The profitability index was used as a criterion for the initial selection of projects for the portfolio.

One of the mathematical programming methods – linear optimization – was used as a *solution method*.

*Research toolkit* – MS Excel spreadsheet processor and Search for a solution (Solver) add-on.

*The object of the research* was the investment portfolio of a particular company, which, having a certain limited investment budget (monetary units), is considering the possibility of participating in the financing of a number of investment projects. The assessment of the profitability of investing in the proposed projects was made according to the criteria of the total net present value (NPV) of the investment portfolio and the profitability index (PI), which ensure the return on investment. As a result of the preliminary analysis, six projects were selected that are attractive for financing [2]. The assumed conditions for the implementation of projects are shown in Table 1.

Since each investor seeks not only to reach the break-even point but also to get the maximum return on investment, the total net present value of projects is used as an optimization criterion, subject to restrictions on the amount of the budget [3].

**Table 1 – Proposed investment portfolio. Source: developed by the authors**

| Project | I | PV | NPV | PI |
|---|---|---|---|---|
| Project A | -80000 | 95000 | 15000 | 1,19 |
| Project B | -60000 | 79000 | 19000 | 1,32 |
| Project C | -70000 | 112000 | 42000 | 1,6 |
| Project D | -100000 | 145000 | 45000 | 1,45 |
| Project E | -40000 | 52000 | 12000 | 1,3 |
| Project F | -110000 | 126500 | 16500 | 1,15 |

The mathematical model of the problem in the proposed setting has the form.

Objective function – the total net present value of projects: $A*X \rightarrow max$,

Restrictions: $C * X \leq B$, $Xk \geq 0$ $(k = 1; n)$,

where $A$ is a matrix of coefficients for variables of the objective function;

$X$ is the vector of variables of the objective function;

$C$ is a coefficient of the constraint function;

$B$ is the vector of restrictions.

If we designate project *"A"* through $X1$, project *"B"* through $X2$, etc., then the objective function of the task can be formulated in a vector form:

$$\max NPV = \begin{vmatrix} 15000 \\ 19000 \\ 42000 \\ 45000 \\ 12000 \\ 16500 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \end{vmatrix}$$

In terms of investment, the budget of the firm is limited to a certain amount (monetary units). Consequently, the total initial costs for the implementation of projects cannot exceed this amount. This condition defines the constraints for this task:

$$\begin{vmatrix} 80000 \\ 60000 \\ 70000 \\ 100000 \\ 40000 \\ 110000 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \end{vmatrix} \leq \text{budget amounts}$$

In addition, the number of projects cannot be negative, and also each project cannot be implemented more than once, that is: $0 \le Xk \le 1$ (k = 1; 6)

The model implementation in the MS EXCEL environment is shown in Figure 1.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | ***Отбор проектов в условиях ограниченного бюджета*** | | | | |
| 2 | | | | | | |
| 3 | **List of projects** | **Objective function coefficients** | **Constraint function coefficients** | **Objective function** | **Constraint function** | **Objective function variables** |
| 4 | | *A* | *C* | | | *Xi* |
| 5 | Project A (X1) | 15000 | 80000 | 0 | 0 | 0 |
| 6 | Project B (X2) | 19000 | 60000 | 19000 | 60000 | 1 |
| 7 | Project C (X3) | 42000 | 70000 | 42000 | 70000 | 1 |
| 8 | Project D (X4) | 45000 | 100000 | 45000 | 100000 | 1 |
| 9 | Project E (X5) | 12000 | 40000 | 6000 | 20000 | 0,5 |
| 10 | Project F (X6) | 16500 | 120000 | 0 | 0 | 0 |
| 11 | | | | | | |
| 12 | max NPV | | | **112000** | | |
| 13 | Budget | | | | **250000** | |

**Figure 1 – Model implementation in the MS EXCEL environment**

Source: developed by the authors *Compiled by the authors.*

Cells F5: F10 contain the values of variables unknown to X (initially, they are set to zero).

In cells D5: D10, the values of the terms of the objective function are calculated.

In cells E5: E10, the terms of the constraints are calculated.

Cell D12 is the total NPV of the optimal investment portfolio.

Cell E13 is the investment budget of the firm (250,000 monetary units accepted).

From the solution given with the help of the Search for solution MS Excel add-on, it follows that with this amount of the investment budget of the firm, the maximum possible value of NPV = 112,000 monetary units. For this, it is necessary to implement 0.5 projects "E", as well as projects "B", "C", "D".

More often, the project cannot be implemented in parts, or investment objects are not subject to fragmentation (buildings, personnel, etc.). Then it is advisable to use integer optimization. For this, a constraint of the form should be added to the developed model: $Xk = (0,1)\ (k = 1; 6)$. In this case, the optimal portfolio will include projects "B", "C", "D", and the total NPV will be 106,000 monetary units.

As a result of the research, we can conclude that the imposition of integer constraints changed the value of the objective function downward. In the general case, the introduction of additional restrictions always leads to a decrease in the optimization effect.

The final decision on the formation of the optimal investment portfolio, in any case, remains to the specialists, and the results of the investment analysis are only the basis for further careful study of various aspects of the project.

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